ESMI 2022 - Network Analysis

6/6/2022

Agenda:
- Introductions
- Intro to applied math program
- Brief intro to networks
- Linear algebra

Intro to Applied Math Program

Topic: Analysis of networks - emphasis on social networks

Overall agenda: intro linear algebra
intro to graphs / networks
graphs/networks in python

Techniques we learn will serve as intro to machine learning

Final project: open-ended
based on personal preference

First two weeks: lots of new network material
weeks 3-9: M professional development
W new network material
F group work time and office hours

Intro to Networks

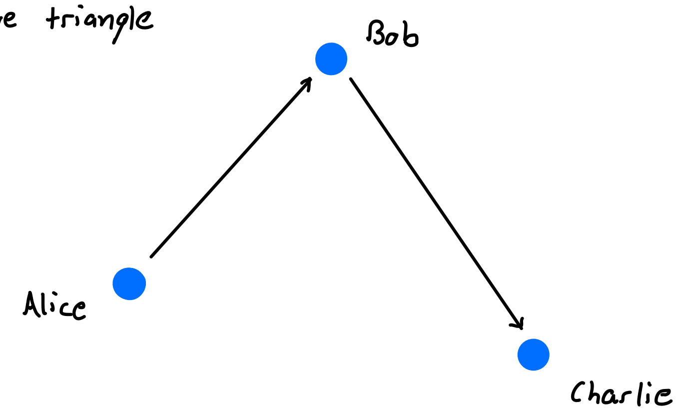
A network is a set of nodes connected by edges. (Also called a graph.)

Nodes correspond to physical things (e.g. people, animals, websites, phones, street corners, ...)

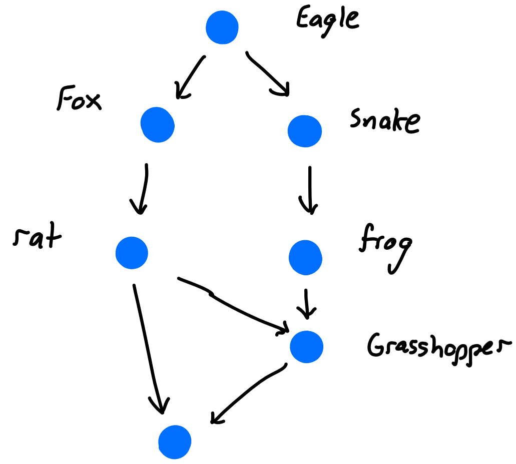
Edges correspond to connections between nodes (e.g. friends on FB, links to other websites, who eats who in a food chain)

Graps can be used for visualization or analysis or understanding something about a group of things

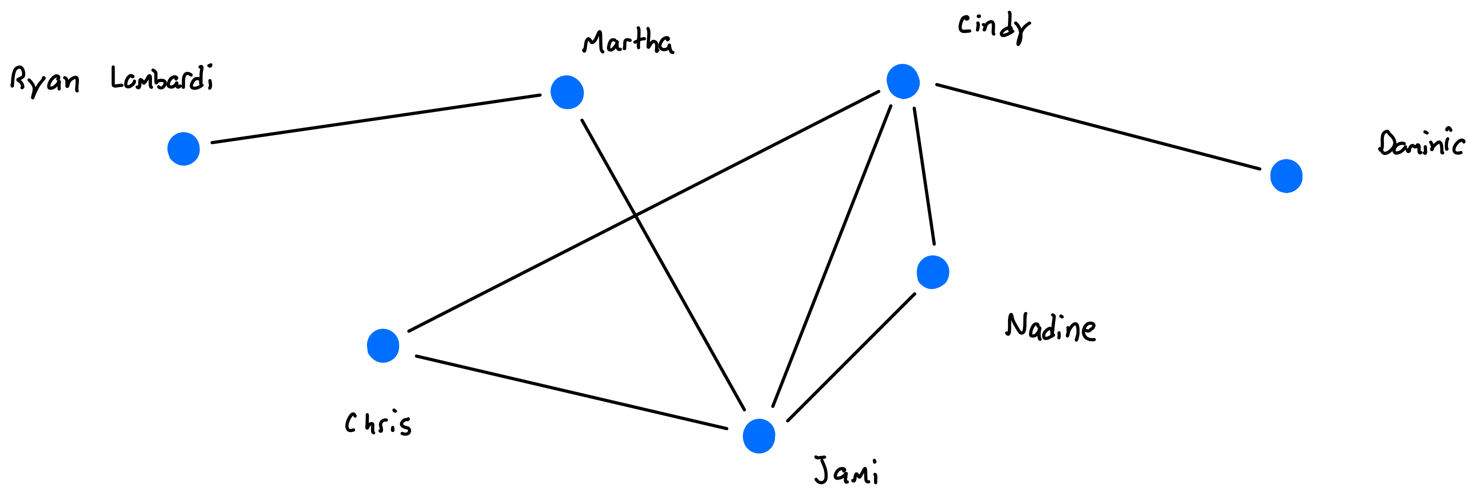
Ex) Love triangle



Ex) Food chain



Ex) Tracking COVID



Networks can help us solve the following question: If Ryan Lombardi gets COVID, how likely is it that Dominic also gets COVID?

In order to use math to work with graphs, we need some linear algebra.

Linear Algebra

Linear algebra is a branch of math that studies systems of equations

For example

$$x + 4y = 9$$
$$x - 2y = -3$$

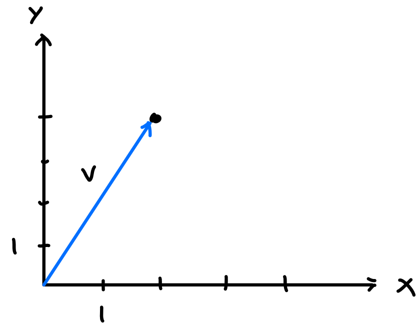
solution: $x = 1$
$y = 2$

but linear algebra can also be used to analyze networks

The linear algebra tools we use to analyze these systems have physical interpretations

Basic Linear Algebra components

- **Vectors** : math quantity that has direction and magnitude
  - Vectors in 2-dimensional space ($\mathbb{R}^2$)

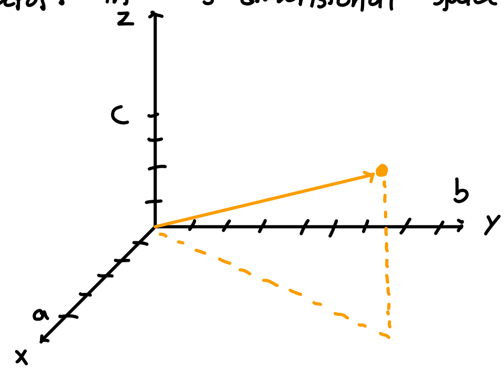


$$V = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \begin{matrix} \leftarrow \text{first component} \\ \leftarrow \text{second component} \end{matrix}$$

magnitude (length) :

$$\|v\| = \sqrt{2^2 + 4^2} = \sqrt{20}$$

  - Vectors in 3-dimensional space ($\mathbb{R}^3$)



$$V = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

  - Vectors in n-dimensional space ($\mathbb{R}^n$)

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix}$$

impossible to draw unfortunately...

- **Vector operations** Addition - requires vectors to be the same size

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} \qquad V_1 + V_2 = \begin{bmatrix} 1+3 \\ 2-1 \\ 4+0 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 4 \end{bmatrix}$$

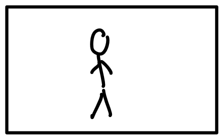
Scalar multiplication - scale vector by a number

$$V_1 = \begin{bmatrix} 2 \\ 8 \\ 9 \\ 0 \end{bmatrix} \qquad 2 \cdot V_1 = \begin{bmatrix} 2 \cdot 2 \\ 2 \cdot 8 \\ 2 \cdot 9 \\ 2 \cdot 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \\ 18 \\ 0 \end{bmatrix}$$

- **Matrix** - math quantity that generalizes vectors
  - can think of a matrix as a table of numbers or vectors placed side by side
  - can encode physical concepts

ex: image

200 pixels



300 pixels

each of the 200×300 pixels have their own Red Green and Blue (RGB) values

0.5
0.6

$$R = \begin{bmatrix} \cdot & \cdot & \cdot & \cdots \\ \vdots & & & \\ & & & \end{bmatrix} \qquad G = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \qquad B = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

R, G, B all have 200 rows and 300 columns → 60,000 numbers in each
=> how many #'s to represent the image? 180,000

ex: Linear system

$$\begin{aligned} 5x + 2y + 1z &= 0 \\ 1x + 1y + 1z &= 0 \\ 2x + \tfrac{1}{2}y + 2z &= 2 \end{aligned} \quad \Rightarrow \quad \left[\begin{array}{ccc|c} 5 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 1/2 & 1 & 2 \end{array}\right]$$

"Augmented matrix"

can use "row reduction" to solve for x, y, z.

ex: Network → can be represented by an "adjacency matrix"
computers cannot understand pictures of networks → represent with matrix

General matrix

start from zero

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1m} \\ A_{21} & A_{22} & & & A_{2m} \\ A_{31} & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & & A_{nm} \end{bmatrix}$$

A_{ij} : "entries" in the matrix, all numbers
n : number of rows
m : number of columns

how many entries in the above matrix? $n \cdot m$ entries.

Size of this matrix: $n \times m$

- Matrix Operations

- Addition (matrices must be the same size) - add entries together

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ 2 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -1 & 2 \\ -1 & 3 & -1 \\ -1 & 10 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 4 & 0 \\ 0 & 11 & -1 \end{bmatrix}$$

- Scalar multiplication - multiply each entry by a scalar

$$2 \cdot \begin{bmatrix} 3 & 8 \\ 2 & -1 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 16 \\ 4 & -2 \\ 10 & -4 \end{bmatrix}$$

- Matrix-matrix multiplication - multiply two matrices together
  requirement: matrix 1 size $m \times n$
  matrix 2 size $n \times p$
  result: matrix of size $m \times p$

will see example in next class...