Name: ______________________________ **ESMI Applied Math**

Due: 06/13 **Worksheet 5**

Problem 1. Compute the matrix-matrix product: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 7 & -3 \\ 3 & 4 & -2 & 0 \end{bmatrix}$.

$$\begin{bmatrix} 1+6 & 2+8 & 7-4 & -3+0 \\ 3+12 & 6+16 & 21-8 & -9+0 \\ -1+0 & -2+0 & -7+0 & 3+0 \end{bmatrix} = \begin{bmatrix} 7 & 10 & 3 & -3 \\ 15 & 22 & 13 & -9 \\ -1 & -2 & -7 & 3 \end{bmatrix}$$

Problem 2. Compute the eigenvalues and eigenvectors of $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.

eigenvalues:

$$0 = \det\left(\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \det\left(\begin{bmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix}\right)$$

$$= (-5-\lambda)(-2-\lambda) - 4$$

$$= \lambda^2 + 7\lambda + 10 - 4$$

$$= \lambda^2 + 7\lambda + 6$$

$$= (\lambda+6)(\lambda+1)$$

$$\lambda = -6, -1$$

Eigenvector for $\lambda = -1$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left(\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} + 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad \Rightarrow V = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$$0 = -4v_1 + 2v_2$$

$$v_1 = \tfrac{1}{2} v_2$$

Eigenvector for $\lambda = -6$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left(\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} + 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

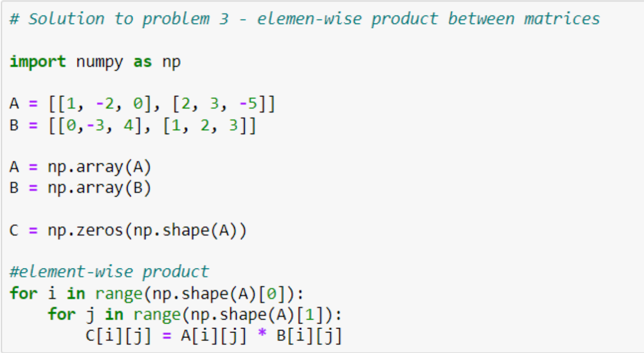
$$0 = v_1 + 2v_2$$

$$v_1 = -2v_2 \qquad \Rightarrow V = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Problem 3. Write code that first stores matrices A and B into python variables then computes the element-wise product between the two matrices. Your answer should use loops.

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -5 \end{bmatrix}, B = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$$

hint: the result should be $C = \begin{bmatrix} 1 \cdot 0 & -2 \cdot -3 & 0 \cdot 4 \\ 2 \cdot 1 & 3 \cdot 2 & -5 \cdot 3 \end{bmatrix}$

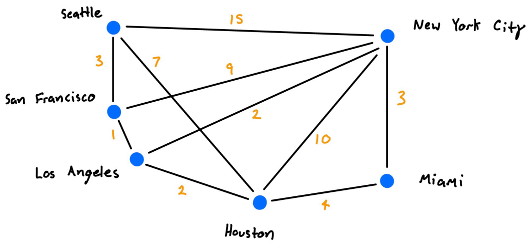


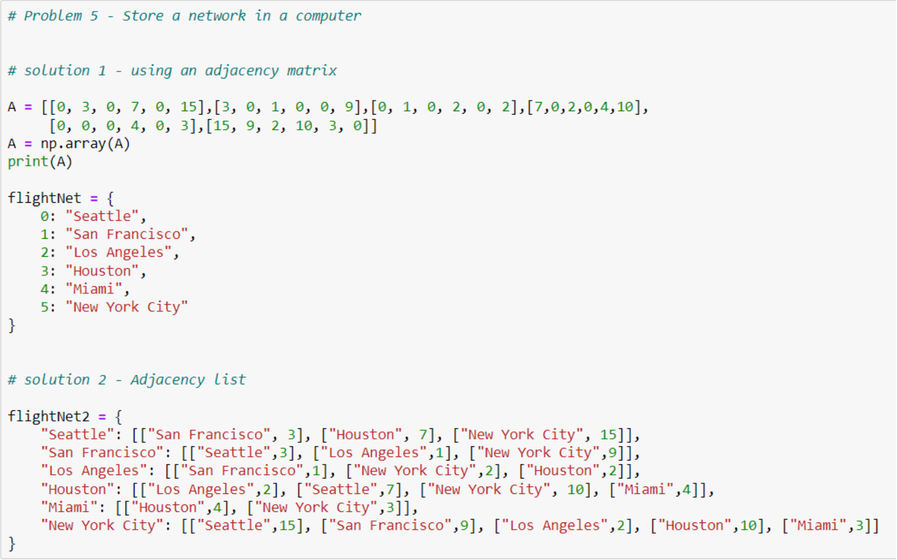
```python
# Solution to problem 3 - elemen-wise product between matrices

import numpy as np

A = [[1, -2, 0], [2, 3, -5]]
B = [[0,-3, 4], [1, 2, 3]]

A = np.array(A)
B = np.array(B)

C = np.zeros(np.shape(A))

#element-wise product
for i in range(np.shape(A)[0]):
    for j in range(np.shape(A)[1]):
        C[i][j] = A[i][j] * B[i][j]
```

Problem 4. Write a python function that takes in two numpy arrays (matrices) as inputs and returns the matrix-matrix product if possible. If the matrix-matrix product is not possible, your function should print "these matrices are not compatible for matrix-matrix multiplication". Code this up first in jupyter notebooks and once you are happy with your answer, please write your function below.

Solution on next worksheet

Problem 5. Think of a way to store all of the information in the following graph into variables in python then do it! Write code in juptyer notebooks first then write your code below. Note: there are many right answers.





```
# Problem 5 - Store a network in a computer


# solution 1 - using an adjacency matrix

A = [[0, 3, 0, 7, 0, 15],[3, 0, 1, 0, 0, 9],[0, 1, 0, 2, 0, 2],[7,0,2,0,4,10],
     [0, 0, 0, 4, 0, 3],[15, 9, 2, 10, 3, 0]]
A = np.array(A)
print(A)

flightNet = {
    0: "Seattle",
    1: "San Francisco",
    2: "Los Angeles",
    3: "Houston",
    4: "Miami",
    5: "New York City"
}


# solution 2 - Adjacency list

flightNet2 = {
    "Seattle": [["San Francisco", 3], ["Houston", 7], ["New York City", 15]],
    "San Francisco": [["Seattle",3], ["Los Angeles",1], ["New York City",9]],
    "Los Angeles": [["San Francisco",1], ["New York City",2], ["Houston",2]],
    "Houston": [["Los Angeles",2], ["Seattle",7], ["New York City", 10], ["Miami",4]],
    "Miami": [["Houston",4], ["New York City",3]],
    "New York City": [["Seattle",15], ["San Francisco",9], ["Los Angeles",2], ["Houston",10], ["Miami",3]]
}
```