Name: ____________________ **ESMI Applied Math**
Due: 06/08 **Worksheet 2**

Problem 1. Compute the dot product between the following vectors or state if the dot product cannot be computed.

(a) $u = \begin{bmatrix} -2 \\ -7 \\ 1/2 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 10 \\ 2 \end{bmatrix}$

$u \cdot v = 0 - 70 + 1$
$= -69$

(b) $u = \begin{bmatrix} -2 \\ -7 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 10 \\ 2 \end{bmatrix}$

not possible because the vectors are not the same size.

Problem 2. Compute the matrix-matrix product between the following matrices or state if you cannot compute the matrix-matrix product.

(a) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} -2 & 17 & 15 \\ -7 & -10 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 17 & 15 \\ -7 & -10 & 2 \end{bmatrix}$

Not possible since
L : 2×3
R : 2×3

Problem 3. Compute the matrix-matrix product B^2 when $B = \begin{bmatrix} 0 & -1 & -2 \\ 1 & -10 & 3 \\ 1 & 0 & -1 \end{bmatrix}$

$$B^2 = \begin{bmatrix} 0 & -1 & -2 \\ 1 & -10 & 3 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & -2 \\ 1 & -10 & 3 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0-1-2 & 0+10+0 & -1 \\ 0-10+3 & 99 & -35 \\ 0+0-1 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 10 & -1 \\ -7 & 99 & -35 \\ -1 & -1 & -1 \end{bmatrix}$$

Problem 4. What condition must a matrix A satisfy in order for you to be able to compute the matrix-matrix product A^2?

The number of rows and columns of A should be the same.

Problem 5. Compute the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$0 = \det\left(\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$
$$= \det\left(\begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}\right)$$
$$= (1-\lambda)(4-\lambda) - 2\cdot 2$$
$$= 4 - \lambda - 4\lambda + \lambda^2 - 4$$
$$= \lambda^2 - 5\lambda$$
$$= \lambda(\lambda - 5)$$

eigenvalues: $\lambda = 0, 5$

eigenvector for $\lambda = 0$:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = (A - \lambda I)v$$
$$= \left(\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$\Rightarrow \quad 0 = 1v_1 + 2v_2$

$v_1 = -2v_2$

set $v_2 = 1$

$\Rightarrow \quad v = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

eigenvectors for $\lambda = 5$:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left(\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$0 = -4v_1 + 2v_2$

$v_1 = \frac{1}{2} v_2$

$v = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$